Valuing a Firm's Capital Structure using Profit Caps, Floors and Bond Default Options

G. Pawlina, M. B. Shackleton and R. Wojakowski
Lancaster University

ESSEC, Oct 2009

1 Capital structure and firm default

- Still debating proper valuation of capital structure 50 years on!
  - Modigliani Miller ('58) [19], Fernandez ('04) [8], Cooper Nyborg ('06) [3]

- Traditionally discount expected cashflow at risk adjusted rate **as if un-contingent,** inadequate when costs of capital are dynamic. Exceptions stem from risk-neutral (**RN**) real options pricing
  - Miles Ezzell ('85) [18], Ruback ('02) [20] amongst others
  - Black Scholes ('73) [1], Cox, Ross, Rubinstein ('79) [4], Trigeorgis ('96) [22]

2 Contingent claim valuation

- Costs of capital are **dynamic** due to embedded debt default option
  - Merton ('74) [17] at finite maturity alone (Ingersoll ('77) [11] bond convertible into equity)
  - McDonald and Siegel ('85) [15] perpetual caps and floors
  - Leland ('94) [13] at lower perpetual default barrier (Leland and Toft ('96) [14] finite bonds subject also to early default)
  - Perraudin and Mella-Barral ('97) [16] strategic service and default (Fan and Sundaresan ('00) [7] possible renegotiation)

3 Empirical papers

- Recent papers that apply and compare empirical model performance
  - Graham ('00) [10]
  - Kemsley and Nissim ('02) [12]
  - Vassalou and Xing ('04) [23]
  - Eom, Helwege and Huang ('04) [6]
  - Bris, Welch and Zhu ('06) [2]

4 This paper

- Finite maturity debt with periodic refinancing (as yet, non strategic)

- Model contingent dividends, coupons, debt repayment and **frictions** explicitly

- **Taxes** (θ, Θ), **default (**λ) **and distress (**Λ) as **lump sums or cashflows**
  - Allocate **cost of capital** to each through explicit boundary conditions
  - Use **new solution** class to complement Merton ('74) [17] payoffs
  - Finite **caps** and **floors** on uncertain **flows**

5 Without frictions or flows

- In a risk-neutral framework, firm profit/earnings/cash flow P_t and total asset value A_t follow RN GBM (δ is the constant asset yield and payout)

$$\frac{dP_t}{P_t} = (r - \delta)\, dt + \sigma dW_t : A_0 = E_0^Q \int_0^\infty e^{-rt} P_t dt = \frac{1}{\delta} P_0$$

- Merton model with face value X and maturity τ : Equity is a call on assets and zero coupon Bond is risk free but short a default put

$$\begin{aligned} E_0 &= e^{-r\tau} E_0^Q \left[(A_\tau - X)^+ \right] = c\,(A_0, X, \tau) \\ B_0 &= e^{-r\tau} E_0^Q \left[\min\,(A_\tau, X) \right] = X e^{-r\tau} - f\,(A_0, X, \tau) \end{aligned}$$

- Black Scholes put/call are functions of two terms with elasticities $1, 0$

$$f(A_0, X, \tau) = Xe^{-rt}N(-d_0) - A_0e^{-\delta t}N(-d_1)$$

$$c(A_0, X, \tau) = A_0e^{-\delta t}N(d_1) - Xe^{-rt}N(d_0)$$

- Traditional BS d_2 replaced by $d_{\beta=0}$ for reasons that will become clear

$$d_{\beta=1,0}(A_0, X, \tau) = \frac{\ln A_0 - \ln X + \left(r - \delta + \left(\beta - \frac{1}{2}\right)\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$$

6 With frictions but without any flows

- On refinancing at time τ, **lump sum** proportional taxes θ and bankruptcy costs λ are levied if $A_\tau \gtrless X$, i.e. a fraction of equity call and default put

$$\text{Tax at } \tau = \theta \max\left(A_\tau - X, 0\right) : \text{B.Costs at } \tau = \lambda \max\left(X - A_\tau, 0\right)$$

- These are minimised* wrt X

$$\begin{aligned} 0 &= \frac{\partial}{\partial X}\left(\theta c\left(A_0, X, \tau\right) + \lambda f\left(A_0, X, \tau\right)\right) \\ &= e^{-r\tau}\left(-\theta N\left(d_0\right) + \lambda N\left(-d_0\right)\right) \\ N\left(-d_0^*\right) &= \frac{\theta}{\lambda + \theta} \qquad \ln \frac{X^*}{A_0} = \left(r - \delta - \frac{1}{2}\sigma^2\right)\tau + \sigma\sqrt{\tau} N^{-1}\left(\frac{\theta}{\lambda + \theta}\right) \end{aligned}$$

- Optimal RN prob. of default $N\left(-d_0^*\right)$ set to $\frac{\theta}{\lambda+\theta}$ (solvency $N\left(d_0^*\right) = \frac{\lambda}{\lambda+\theta}$)

- As a fraction of $A_0 e^{-\delta\tau}$, the frictions increase with all of $\theta, \lambda, \sigma\sqrt{\tau}$. At future time τ, re-financing allows capital structure to be **re-optimised**

$$
\begin{aligned}
N\left(d_1^*\right) &\approx N\left(d_0^*\right) + \sigma\sqrt{\tau} n\left(d_0^*\right) \\
\text{Minimised frictions} &\approx A_0 e^{-\delta\tau}\left(\theta+\lambda\right)\sigma\sqrt{\tau} n\left(d_0^*\right)
\end{aligned}
$$

? Interim $t \in \{0, \tau\}$ cash flows δA_t and value $A_0\left(1 - e^{-\delta\tau}\right)$

? Managerial incentives and costs of resetting the bond contract

? How frequently to reset

7 Without frictions but with flows: Equity

- For $t \in \{0, \tau\}$, partition firm profit/earnings to equity and debt around coupon cost rate and make dividends contingent on current profit rate $P_t \lesseqgtr K$

$$P_t = \max(P_t - K, 0) + \min(P_t, K)$$

- Equity is a **cap** $C(P_0, K, \tau)$ on profits at coupon level K plus **call** $c(A_0, X, \tau)$ on residual over face value X

$$\begin{aligned} E_0 &= \int_0^\tau e^{-rt} E_0^Q \left[(P_t - K)^+ \right] dt + e^{-r\tau} E_0^Q \left[(A_\tau - X)^+ \right] \\ \text{Cap } C(P_0, K, \tau) &= \int_0^\tau e^{-rt} E_0^Q \left[(P_t - K)^+ \right] dt \\ E_0 &= C(P_0, K, \tau) + c(A_0, X, \tau). \end{aligned}$$

8 Without frictions but with flows: Debt

- Bond is annuity less **floor** $F(P_0, K, \tau)$, plus risk free face value less default **put** $f(A_0, X, \tau)$

$$\begin{aligned} B_0 &= \int_0^\tau e^{-rt} E_0^Q [\min(P_t, K)]\, dt + e^{-r\tau} E_0^Q [\min(A_\tau, X)] \\ \text{Floor } F(P_0, K, \tau) &= \int_0^\tau e^{-rt} E_0^Q [\min(P_t, K)]\, dt \\ B_0 &= \frac{K}{r}(1 - e^{-r\tau}) - F(P_0, K, \tau) + X e^{-r\tau} - f(A_0, X, \tau)\,. \end{aligned}$$

- Without frictions parity (cap $-$ floor $=$ swap) ensures $E_0 + B_0 = A_0$

9 Debt yield spreads

- *YTMs* solve for y; not a true rate of return but often treated as such

$$B_0 = \frac{K}{y}\left(1 - e^{-y\tau}\right) + Xe^{-y\tau} \qquad \text{or} \qquad y = \frac{1}{\tau}\ln\frac{X - \frac{K}{y}}{B_0 - \frac{K}{y}}$$

- *Yield spread:* the excess of y over risk free e.g. shown for $(A_0, X, P_0, K, \sigma, r, \delta)$ $= (1250, 1000, 125, 100, 20\%, 10\%, 10\%)$ and τ to 32 years

- A richer structure, nice asymptote but without frictions still problem at short end if $P_0 > K$

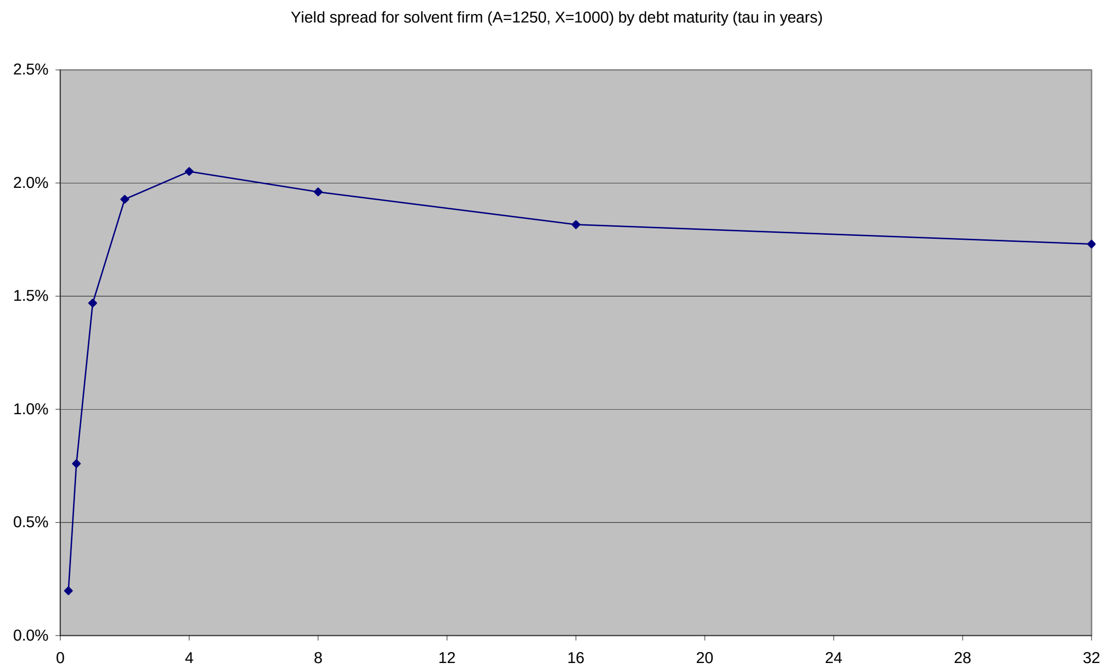

Figure 1: Yield spread $y - r$ against maturity τ.

10 Separating distress and default

- Bond (near par if $K \approx yX$) is subject to **two** risks with **different** costs of capital

$$B_0 = X - F(P_0, K, \tau) - f(A_0, X, \tau)$$

- **Default** is a one time event that occurs, if at all, at refinancing τ
  - an asset **put** $f(A_0, X, \tau)$ at face value X with RN prob. $N(-d_0(\tau))$

- **Distress** (without subsequent recourse) can occur at any time up to τ
  - a non-cumulative profit **floor** $F(P_0, K, \tau)$ with RN $\int_0^\tau N(-d_0(t))\, e^{-rt} dt$

11 Distress in Leland et al.

- Leland [13], Goldstein et al. [9] (Eq 24); equity is PV of profit net of **full coupon payment** until an optimal bankruptcy stopping time $\underline{t}$ and threshold $\underline{P} = P_{\underline{t}}$

$$\text{Equity} = E_0^Q \int_0^{\underline{t}} e^{-rt} (P_t - K)\, dt.$$

- Cashflow within the integral can go **negative** if $P_t < K$; **distress** where equity holders fund coupons in full until immediate default time $\underline{t}$

- Time $\underline{t}$ is defined by contact with optimal default barrier below coupon rate $\underline{P} < K$, first/second order conditions on $\underline{P}$ ensure value matching/smooth pasting of equity to zero

12 With frictions and flows

- Frictional model contingent on interim **and** refinancing level

λ Cost of refinancing in **default** at τ proportional to **put** $\lambda f\left(A_0, X, \tau\right)$

Λ Cost of **distress** (postponing default) to τ proportional to **floor** $\Lambda F\left(P_0, K, \tau\right)$

θ Residual "**tax**" at τ (managerial incentives) at τ proportional to **call** $\theta c\left(A_0, X, \tau\right)$

Θ **Net profit tax** (if positive, without loss carryforward) **cap** $\Theta C\left(P_0, K, \tau\right)$

13 Total frictions and net firm value

- Total taxes and incentives T_0, distress/default D_0 and their sum total frictional losses L_0

$$
\begin{aligned}
T_0 &= \Theta C(P_0, K, \tau) + \theta c(A_0, X, \tau) \\
D_0 &= \Lambda F(P_0, K, \tau) + \lambda f(A_0, X, \tau) \\
L_0 &= D_0 + T_0
\end{aligned}
$$

- From exogenous A_0, also define net firm value V_0

$$
\begin{aligned}
A_0 &= L_0 + V_0 \\
V_0 &= B_0 + E_0.
\end{aligned}
$$

14 Perpetual flows ($\tau \longrightarrow \infty$) and frictions

- McDonald and Siegel ('85) [15], Dixit and Pindyck ('94) [5] evaluated the **perpetual** cap and floor values

$$
\begin{aligned}
P_0 &> K : C\left(P_0, K, \infty\right) = \frac{P_0}{\delta} - \frac{K}{r} + A\left(b\right) P_0^b \\
P_0 &< K : C\left(P_0, K, \infty\right) = A\left(a\right) P_0^a \\
a, b &= \frac{1}{2} - \frac{r-\delta}{\sigma^2} \pm \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}
\end{aligned}
$$

- Value the **tax** and **distress** costs (default is irrelevant), minimise total frictions wrt K

$$\begin{aligned}\text{Tax} &= \Theta C\left(P_0, K, \infty\right) : \text{Distress} = \Lambda F\left(P_0, K, \infty\right) \\ \min_K \text{Frictions} &\Rightarrow \frac{\partial\left(\Theta C\left(P_0, K, \infty\right) + \Lambda F\left(P_0, K, \infty\right)\right)}{\partial K} = 0\end{aligned}$$

- The kappas (derivs wrt K) $\kappa_{C,F}$ are available and yield

$$\begin{aligned}\Theta\kappa_C + \Lambda\kappa_F &= \left(\Theta + \Lambda\right)\left(\frac{P_0}{K}\right)^b \frac{1-b}{|b-a|}\left(\frac{a}{r} - \frac{a-1}{\delta}\right) - \frac{\Theta}{r} \\ \frac{\Theta}{\Theta + \Lambda} &= \left(\frac{P_0}{K^*}\right)^b \frac{1-b}{|b-a|}\frac{a\left(\delta - r\right) + r}{\delta}\end{aligned}$$

15 Optimal perpetual coupon

- This is linked to a RN expected, discounted value of a dollar at the distress time $P_{\underline{t}'} = K^*$ if reached at all. Optimal "probability" (of $\underline{t}'$ distress) and coupon K^* set as a function of $\frac{\Theta}{\Theta+\Lambda}$

$$\begin{aligned} \left(\frac{P_0}{K^*}\right)^b &= E_0^Q\left[e^{-r\underline{t}'}\right] \\ \frac{K^*}{P_0} &= \left(\frac{\Theta}{\Theta+\Lambda}\frac{|b-a|}{1-b}\frac{\delta}{a(\delta-r)+r}\right)^{-\frac{1}{b}} \end{aligned}$$

- Since $b < 0$, the optimal coupon K^* **decreases with distress** costs Λ

16 Finite cap and floor formulae

- Shackleton and Wojakowski ('07) [21] extend McDonald and Siegel ('85) [15] to finite case $\tau < \infty$

$$
\begin{aligned}
C(P_0, K, \tau) &= \frac{P_0}{\delta}\left(\mathbf{1} - e^{-\delta\tau} N(d_1)\right) - \frac{K}{r}\left(\mathbf{1} - e^{-r\tau} N(d_0)\right) + \\
&\quad A(b) P_0^b (\mathbf{1} - N(d_b)) - A(a) P_0^a (\mathbf{1} - N(d_a)) \\
d_\beta(P_0, K, \tau) &= \frac{\ln P_0 - \ln K + \left(r - \delta + \left(\beta - \frac{1}{2}\right)\sigma^2\right)\tau}{\sigma\sqrt{\tau}}
\end{aligned}
$$

- $\mathbf{1} = \mathbf{1}_{P_0 \geqslant K}$ is an indicator function that is one if $P_0 \geqslant K$, zero otherwise; new RN probs $N(d_a), N(d_b)$ are calculated for $\beta = a > 1, b < 0$ elasticities other than those traditionally used $(1, 0)$.

17 Cap floor parity condition

- Derive floor from a condition that says; Cap $-$ Floor $=$ Swap

$$C\left(P_0, K, \tau\right) - F\left(P_0, K, \tau\right) = \frac{P_0}{\delta}(1 - e^{-\delta\tau}) - \frac{K}{r}(1 - e^{-r\tau})$$

- Four constants $A\left(\beta\right)$ from perpetual solution; see Dixit and Pindyck ('94) [5]

$$\begin{aligned} A\left(a\right) &= \frac{K^{1-a}}{\left|a-b\right|}\left(\frac{b}{r} - \frac{b-1}{\delta}\right) \qquad A\left(0\right) = \frac{K}{r} \\ A\left(b\right) &= \frac{K^{1-b}}{\left|b-a\right|}\left(\frac{a}{r} - \frac{a-1}{\delta}\right) \qquad A\left(1\right) = \frac{1}{\delta} \end{aligned}$$

18 Explicit valuation of frictions

- For the case with finite maturity debt and both flows before and payoffs at τ, the formulae are easy enough to use to produce numerics for the optimal levels

$$\begin{aligned}\text{Frictions} &= \Lambda F(P_0, K, \tau) + \lambda f(A_0, X, \tau) + \Theta C(P_0, K, \tau) + \theta c(A_0, X, \tau) \\ &= \text{Distress floor} + \text{Default put} + \text{Tax cap} + \text{Incentive call}\end{aligned}$$

- Frictions can be minimised wrt K, X and it will be interesting to see resulting condition on K^*/X^* as a function of $\frac{\Theta}{\Theta+\Lambda}, \frac{\theta+\lambda}{\theta}$

- In practice unless the coupon rate is close to a par yield, the tax shield may not be allowable.

19 Summary

- Existing models allow for contingent payoffs at one boundary, maturity or default. Can treat earlier cashflows with exogenous process (not unlevered firm) $A_t = \frac{1}{\delta} P_t$
- Cap and floor technology allows contingent sharing for all times to τ
- Tackle four frictions separately and additatively; valuation in closed form and optimise over a rich structure K and X (also τ which has been fixed)
- Component costs of capital can be calculated, compare to other models and also use to estimate frictions empirically

References

[1] Fischer Black and Myron Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(May–June):637–659, 1973.

[2] Arturo Bris, Ivo Welch, and Ning Zhu. The costs of bankruptcy. *Journal of Finance*, 61(3):1253–1303, 2006.

[3] Ian A. Cooper and Kjell G. Nyborg. The value of tax shields IS equal to the present value of tax shields. *Journal of Financial Economics*, 81(1):215–225, 2006.

[4] John C. Cox, Stephen A. Ross, and Mark Rubinstein. Option pricing: A simplified approach. *Journal of Financial Economics*, 7:229–263, 1979.

[5] Avinash K. Dixit and Robert S. Pindyck. *Investment under Uncertainty*. Princeton University Press, 1994.

[6] Young Ho Eom, Jean Helwege, and Jingzhi Huang. Structural models of corporate bond pricing: an empirical analysis. *Review of Financial Studies*, 17:499–544, 2004.

[7] Hua Fan and Suresh Sundaresan. Debt valuation, renegotiation and optimal dividend policy. *Review of Financial Studies*, 13:1057–1099, 2000.

[8] Pablo Fernandez. The value of tax shields is NOT equal to the present value of tax shields. *Journal of Financial Economics*, 73(1):145–165, 2004.

[9] Robert Goldstein, , Nengjiu Ju, and Hayne E. Leland. An EBIT-based model of dynamic capital structure. *Journal of Business*, 74:483–512, 2001.

[10] John R. Graham. How big are the tax benefits of debt? *Journal of Finance*, 55(5):1901–1942, 2000.

[11] Jonathon E. Ingersoll. A contingent claims valuation of convertible securities. *Journal of Financial Economics*, 4(3):289–321, 1977.

[12] Deen Kemsley and Doron Nissim. Valuation of the debt tax shield. *Journal of Finance*, 57(5):2045–2073, 2002.

[13] Hayne E. Leland. Corporate debt value, bond covenants and optimal capital structure. *Journal of Finance*, 49(4):1213–1252, 1994.

[14] Hayne E. Leland and Klaus B. Toft. Optimal capital structure, endogenous bankruptcy, and the term stucture of credit risk. *Journal of Finance*, 51(3):987–1019, 1996.

[15] Robert L. McDonald and Daniel R. Siegel. Investment and the valuation of firms when there is an option to shut down. *International Economic Review*, 26(2):331–349, 1985.

[16] Pierre Mella-Barral and William R.M. Perraudin. Strategic debt service. *Journal of Finance*, 52(2):531–556, 1997.

[17] Robert C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29(2):449–470, 1974.

[18] James Miles and Russell Ezzell. Reformulating tax shield valuation. *Journal of Finance*, 40(5):1485–1492, 1985.

[19] Franco Modigliani and Merton H. Miller. The cost of capital, corporation finance and the theory of investment. *American Economic Review*, 48(3):261–297, 1958.

[20] Richard S. Ruback. Capital cash flows: a simple approach to valuing cashflows. *Financial Management*, 31:85–103, 2002.

[21] Mark B. Shackleton and Rafal Wojakowski. Finite maturity caps and floors on continuous flows. *Journal of Economic Dynamics and Control*, 31(12):3843–3859, 2007.

[22] Lenos Trigeorgis. *Real options: Managerial flexibility and strategy in resource allocation*. MIT Press, 1996.

[23] Maria Vassalou and Yuhang Xing. Default risk in equity returns. *Journal of Finance*, 54(2):831–868, 2004.